

# Web Collaboration on Semiformal Mathematical Knowledge

CICM Ph. D. Symposium 2008

Christoph Lange

Jacobs University, Bremen, Germany

Advisor: Michael Kohlhase

July 30, 2008

# Semiformal Mathematical Knowledge

A lot of MKM takes place in the wide area between

- *completely informal and unstructured text*: e. g. blackboard
- *fully formalised knowledge*: e. g. algebraic specification

New theories (definitions, axioms) in development, proof sketches, educational material, . . .

## Example

If the lecturer knows that a proof sketch is valid, he should be able to annotate it as a “proof” and retrieve it when searching for proofs.

# Collaboration on Mathematical Knowledge

Research and education is mostly collaborative:

**Research:** First ideas (e. g. on developing a mathematical model for some domain of application) circulate in a research group  
In a later stage: publish worked-out ideas for peer review  
Large collaborative efforts like Flyspeck

**Education:** Professor, teaching assistants, and smart students create exercises together

In the computer age, people still communicate with pen and paper (well, maybe PDF e-mail attachments. . . )

Create an integrated environment for collaborative editing, browsing, and reviewing.

# A Wiki for Semiformal Mathematical Knowledge

Structuring mathematical knowledge for knowledge management is hard work.

- Solution: semantic markup (“content markup”): Say what symbols/structures mean, not how they look
- Involves thinking – machine should offer support

## Leitmotiv of my research

“How can users be motivated and supported to make the effort of collaboratively structuring mathematical knowledge, what additional knowledge can be inferred from users’ contributions, and how can this again be utilised in order to improve collaboration?”

# State of the Art

- **Semiformal knowledge representation:**

- MathLang, OMDoc: independent of logical foundations, annotate natural language, stepwise formalisation from informal text
- Semantic Web: smart, web-scalable, interlinked metadata, and inference

- **Web collaboration:**

- Wikis: web sites that anyone can edit
- Collaborative features in mathematical applications (Mathematica, Maple, Matlab)

**Semantic wikis** exist but are not suited for MKM: too informal, no support for mathematical knowledge representation (e. g. semantically structured formulæ), no domain knowledge

# Research Methodology

- 1 Gather **use cases**, analyse **requirements** of existing MKM projects w. r. t. collaboration: OpenMath Content Dictionaries, Flyspeck, ...
- 2 Extend/improve/adapt/trim the **knowledge representation** of OMDoc (or OpenMath) to suit an integrated web collaboration platform ( $\rightarrow$  ontology)
- 3 Build a **system prototype** based on this knowledge representation, support selected requirements of above projects
- 4 Deploy system to these projects, collect feedback from **case studies**. Go to (1).

Actually, not that strictly separated ☺

# Expected Contribution

## Core

- A usable platform for **web collaboration** on **semiformal mathematical knowledge**
- An OMDoc-based **knowledge representation** suited for the requirements of web collaboration and embedded into the infrastructure of the semantic web.

## By-products

- **Semantic Web:** mathematics as a quite complex use case pointing out the limits of semantic wikis, extraction of RDF outlines from a complex XML markup
- **MKM:** harnessing the Web 2.0 collaboration potential, contribution MathML 3, OpenMath 3, OMDoc 1.x/2.0

# Roadmap

## Things done

- Ontology of types of math. statements and their interrelations ✓
- SWiM prototype system supporting basic editing, import/export, rendering and browsing of OMDoc and OpenMath documents ✓
- Analysed collaboration requirements for OpenMath CDs, large-scale formalisation (Flyspeck), lecture note authoring ✓

## Work in progress

- Deploying SWiM to OpenMath Society ✈
- Extending ontology by required non-mathematical concepts:
  - rhetorical structures (reuse/integrate SALT/RST) ✈
  - structures of documents and document fragments (SALT) ✈
  - argumentation about mathematical knowledge (reuse DILIGENT) ✈

# Math. Knowledge Representation on the Semantic Web

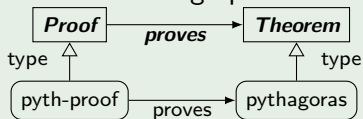
Knowledge stored in OMDoc, but extracting an RDF outline to make it operational:

## Example

An OMDoc document:

```
<omdoc>
  <proof id="pyth-proof"
    for="pythagoras">
    ...
  </proof>
</omdoc>
```

Extracted RDF graph:



```
<pyth-proof, rdf:type, math:Proof>
<pyth-proof, math:proves, pythagoras>
```

Developing ontology based on OMDoc and OpenMath

# Content Dictionaries, Symbols, and Notations



User

- User Page (Administrator)
- Preferences
- Logout
- Theme: [tundra] [soria]

Navigation

- IkeWiki Help
- Recent Changes

Search

Go Search

Edit

- Create Resource
- Create Class
- Create Property
- Create Multimedia
- Create Template
- Delete Resource
- Add Relation
- Remove Relation

System

- Manage Action

Article **arith1** | Discuss | Metadata | Context | Edit | Annotate | History

Identifier: `cd:arith1`

Types: `rdfs:Resource - omo:ContentDictionary - omo:OpenMathConcept - rdfs:Resource`

This CD defines symbols for common arithmetic functions. The symbol to represent the n-ary function to return the least common multiple of its arguments.

### Symbol Definition (plus)

COMMENTED PROPERTY: for all  $a, b \mid a + b = b + a$

FORMAL PROPERTY:  $\forall a, b. (a + b) = (b + a)$

This symbol denotes unary minus, i.e. the additive inverse.

### Symbol Definition (unary\_minus)

COMMENTED PROPERTY: for all  $a \mid a + (-a) = 0$

FORMAL PROPERTY:  $\forall a. (a + - (a)) = 0$

The symbol representing a binary minus function. This is equivalent to adding the additive inverse.

### Symbol Definition (minus)

COMMENTED PROPERTY: for all  $a, b \mid a - b = a + (-b)$

FORMAL PROPERTY:  $\forall a, b. (a - b) = (a + -(b))$

The symbol representing an n-ary multiplication function.

### Symbol Definition (times)

EXAMPLE:  $\begin{pmatrix} 12 & 56 \\ 34 & 78 \end{pmatrix} = \begin{pmatrix} 1922 \\ 4350 \end{pmatrix}$

COMMENTED PROPERTY: for all  $a, b \mid a * 0 = 0$  and  $a * b = a * (b - 1) + a$

References

outgoing

- ikewiki:hasAuthor
- omo:contains

omo:containsSymbol

- arith1+abs
- arith1+divide
- arith1+gcd
- arith1+lcm
- arith1+minus
- arith1+plus
- arith1+power
- arith1+product
- arith1+root
- arith1+sum
- arith1+times
- arith1+unary\_minus

rdf:type

omo:ContentDicto

omo:OpenMathCon

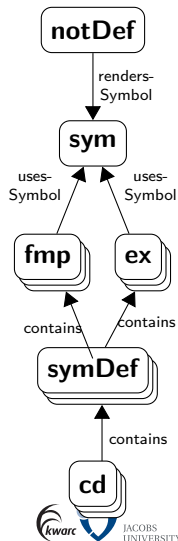
rdfs:Resource

incoming

untyped

Socialise

- Digg this
- Post to del.icio.us
- Post to Furl
- Post to Magnolia
- Post to Yahoo
- Permalink



Demo: <http://swim.kwarc.info>

# Argumentation about Mathematical Knowledge Items

Article Discuss Metadata Context Edit Annotate History

**Issue/Incomprehensible** Alice  
Wed, 14 May 2008 18:00:24 +0100

I don't understand anything. How can this be applied?

Idea Decision

**Idea/ProvideExample** Bob  
Wed, 14 May 2008 18:00:42 +0100

I'd suggest to provide an example that applies this theorem in a concrete domain.

Agree Disagree Decision

**We don't need this.** Cecil  
Wed, 14 May 2008 19:02:16 +0100

Reply

**Great Idea! Let's do that.** Dan  
Wed, 14 May 2008 19:01:52 +0100

Reply

New Issue New Comment

Article Discuss Metadata Context Edit Annotate History

SampleTheorem Languages: [en]

Identifier: SampleTheorem

Types: *odo:Theorem* - *odo:Assertion* - *odo:NonconstitutiveStatement* - *odo:OMDocConcept* - *odo:Statement* - *rdfs:Resource*




There is the following issue with this knowledge item: It was found to be incomprehensible.  
Suggested action: [Create an example for it.](#)

**THEOREM:**  
For every sound and complete calculus, there is ...

Take the survey:  
[tinyurl.com/5qdetd](http://tinyurl.com/5qdetd)

# To Do

## Things to be done ( $\approx$ 1 year left)

- Conduct and evaluate OpenMath case study 
- Deploy SWiM for other case studies (interested?) 
- More inference rules, more usable editing and browsing, ... 
- Write Ph. D. thesis

# Related Work

- Other symposium participants:
  - **Plat $\Omega$ /T $\epsilon$ Xmacs** (M. Wagner): intelligent authoring
  - **panta rhei** (C. Müller): communities of practice in mathematics
  - **TNTBase** (V. Zholudev): distributed versioned database for OMDoc
- Others:
  - **PlanetMath**: mathematical web encyclopedia
  - **Wiki** for formal mathematics: ProofWiki
  - **ActiveMath**: e-learning

# Conclusion

- Web collaboration on semiformal mathematical knowledge ...
  - in a tool that can author and utilise that knowledge
  - for research, development, and education
  - with a semantic wiki for OMDoc and OpenMath
- Experimental approach: collect requirements, develop formal model, test it in a real system in real use cases
- Results so far:
  - semantic web ontology for structures of math. knowledge
  - integrated with rhetorical and argumentation ontology
  - collaborative editor for OMDoc and OpenMath

# Conclusion

- Web collaboration on semiformal mathematical knowledge ...
  - in a tool that can author and utilise that knowledge
  - for research, development, and education
  - with a semantic wiki for OMDoc and OpenMath
- Experimental approach: collect requirements, develop formal model, test it in a real system in real use cases
- Results so far:
  - semantic web ontology for structures of math. knowledge
  - integrated with rhetorical and argumentation ontology
  - collaborative editor for OMDoc and OpenMath

Bringing Web 2.0 to MKM